

SENIOR HIGH MATH LEAGUE
April 24, 2001

GROUP V OPEN DIVISION
TEST A

Unless otherwise stated give exact answers.

SECTION I: ONE POINT EACH

1. Find the quotient when $8x^3 + 1$ is divided by $2x + 1$.
2. Given $f(x) = 5x - 2b$ while $g(x) = 4bx$. If $f(g(1)) = 36$, what is $g(f(1))$?
3. Give the exact value for the following: $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right)$
4. A triangle and square share the same base. The area of the triangle is 10 square units less than the area of the square. Find the height of the triangle in terms of its base b .
5. What is the area of the figure that has the points $(1, 3)$, $(3, 3)$, and $(1, 6)$ as vertices?
6. At the points where all functions listed are defined $\sin(2\theta)$ is equivalent to which of the following expressions?

(a) $2 \cos^2 \theta - 1$

(b) $2 \sin \theta \cos \theta$

(c) $\frac{2 \sin^2 \theta - 1}{-\cos^2 \theta - \sin^2 \theta}$

(d) all of these

(e) none of these

SECTION II: TWO POINTS EACH

1. The sides of a triangle are 3, 5, and 6 in. Find the exact perimeter of a similar triangle whose shortest side is 10 in. The answer should be expressed as one fraction and be reduced.
2. Find the area of a square with diagonals of length 10ft.

3. Evaluate the following expression exactly where the angles are given in degrees.

$$\sin^2(10^\circ) + \sin^2(20^\circ) + \sin^2(30^\circ) + \dots + \sin^2(60^\circ) + \sin^2(70^\circ) + \sin^2(80^\circ)$$

4. A student scored 60% on a 20 question exam. The instructor let him work an additional 20 problems to improve his score. How many must the student get right to raise his grade to 70%?
5. A silo consists of a cylinder with a hemisphere on top. If the height of the cylinder is half of the diameter of the hemisphere and the total volume of the silo is 600 cubic feet, how many cubic feet will the cylinder part of the silo hold?
6. The product of two consecutive even integers is 12 more than the square of the smaller integer. Find both integers.
7. An investor has 100 shares total of two kinds of stock: Dot Com and Big Blue Corp. Dot.Com sells for \$20 per share and Big Blue sells for \$50 per share. If the investor has \$2900 total in stock, how many shares of each does he own?

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GROUP V OPEN DIVISION
TEST B

Unless otherwise stated give exact answers.

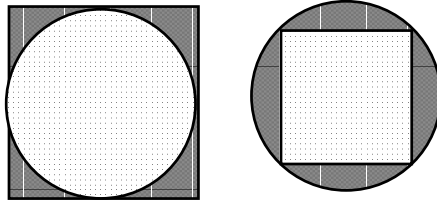
SECTION I: ONE POINT EACH

1. Find the smallest integer x such that $|5 - 2x| \leq 6$.
2. The measure of one of two complementary angles is 27 less than twice the measure of the other. Find the measure in degrees of the smaller of the two angles.
3. True or False: A quadratic equation with integer coefficients may have one real and one complex solution.
4. Find the ordered pair (x,y) with the smallest possible y value that satisfies both $y \geq x^2$ and $y \geq x + 2$.
5. 210 feet of fence is used to make a rectangular pen whose length is twice the width. What is the area of the pen?
6. What is the maximum number of obtuse angles contained within any parallelogram?

SECTION II: TWO POINTS EACH

1. Solve the equation $2\sin 2\theta - 2\sqrt{3}\cos\theta = 0$ on the interval $0 \leq \theta < 2\pi$. Express your answers as exact values in radians and in increasing order of magnitude.
2. An aquarium has a length of 15 in. and a width of 11 in. A rock put into the aquarium causes the water level to rise by 2 in. The rock is completely submerged. What is the volume of the rock? Express your answer in cubic inches.
3. A number n is added to the numerator and the denominator of the fraction $3/5$ and the result is a fraction with value $4/5$. Find the number n .

4. A circle of radius 2 meters is inscribed within a square and a square is inscribed within another circle of radius 2 meters (see diagram). Find the exact sum of the areas of the two darkly shaded regions.



5. A college student is going home for Thanksgiving. The student takes a helicopter from the dorm to the airport and a plane from the airport home. The helicopter averages 70 mph and the plane travels at a rate of 500 mph. The total time spent on the helicopter and plane was 2 hours and the total distance travelled by helicopter and plane is 785 miles. Find the distance from the dorm to the airport.
6. Suppose a 3 letter code word is formed using the first 8 letters of the alphabet. How many code words can be formed if no letter can be repeated?
7. A survey of 420 people showed that 250 owned an automobile, 150 owned a house, and 100 owned both an automobile and a house.
- To the nearest percent, what percent of people owned neither an automobile or a house?
 - To the nearest percent, what percent of people owned an automobile only?

SCHOOL NAME _____

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GROUP V OPEN DIVISION
Form A

Answer

SECTION I: ONE POINT EACH
EACH

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

_____ shares of Dot Com

_____ shares of Big Blue

SECTION II: TWO POINTS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. a) _____
b) _____

SCHOOL NAME _____

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GROUP V OPEN DIVISION
Form B

Answer

SECTION I: ONE POINT EACH
EACH

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

SECTION II: TWO POINTS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. a) _____
b) _____