

SENIOR HIGH MATH LEAGUE
April 2001

GROUP II Emphasis on ALGEBRA II

TEST A

Unless otherwise stated, all answers should be given as a simplified fraction.

SECTION I: ONE POINT EACH

1. Simplify $\left(\frac{2x^2 - 7x - 4}{20 - x - x^2}\right) \div \left(\frac{2x^2 - 9x - 5}{x^2 - 25}\right)$ as much as possible.
2. Factor completely using integer coefficients: $5x^3 - 125x$
3. Suppose $f(x) = x^2 + bx - 3$ and $f(2) = -9$. Find b .
4. Given $f(x) = 5x - 2b$ while $g(x) = 4bx$. If $f(g(1)) = 36$, what is $g(f(1))$?

SECTION II: TWO POINTS EACH

1. Give the phrase that best describes each of the relations in the xy -plane. Give your answer as either "NOT A FUNCTION", "ONLY A FUNCTION", or "A ONE TO ONE FUNCTION".
 - (a) $x = 9$
 - (b) $y = 9x^2$
 - (c) $y = \frac{1}{x}$
 - (d) $x^2 + y^2 = 9$
2. Express the solution as a single inequality: $x - 1 \leq 2x + 4 \leq 3x - 1$
3. A student scored 60% on a 20 question exam. The instructor let him work an additional 20 problems to improve his score. How many must the student get right to raise his grade to 70%?
4. Two inlet pipes can fill a water basin in 10 hours and 12 hours respectively when open individually. If an outlet pipe can empty the basin alone in 9 hours, how long would it take to fill the basin if all three pipes are open simultaneously? (Remember to express solution as a fraction.)
5. The product of two consecutive even integers is 12 more than the square of the smaller integer. Find both integers.
6. Find the equation of a line passing through $(2, 5)$ and parallel to the line given by $4x - y = 7$. Express your answer in the form $y = mx + b$.
7. If $x^2 - 6x + c = 0$ is to have two real roots, c has to be less than what value?
8. Find the equation of a line perpendicular to $y = 4$ and passing through the midpoint of the line segment connecting $(2,5)$ and $(8,-2)$.

Unless otherwise stated, all answers should be given as a simplified fraction.

SECTION I: ONE POINT EACH

- Phone plan A charges 7 cents a minute for all long distance calls. Phone plan B charges a \$5 fixed monthly charge but then just 5 cents a minute for all long distance calls. How many minutes of phone calls must I make in order to make Phone plan B as cost effective as plan A?
- Let $f(x) = \sqrt[4]{x-2}$ and $g(x) = x^{12} + 2$.
 - Find $f(g(-1))$.
 - Find $f(g(x))$.
- Find all integers x satisfying $4x > -x + 5 > 3x - 4$.
- True or False: A quadratic equation with integer coefficients may have one real and one complex solution.

SECTION II: TWO POINTS EACH

- Solve for all real values of x :

$$\sqrt[3]{x} = \sqrt{2\sqrt[3]{x} + 3}$$
- Solve the given system of equations and express your solution as an ordered pair (x,y) .

$$2x + 3y = 8$$

$$3x - 2y = -1$$
- A survey of 420 people showed that 250 owned an automobile, 150 owned a house, and 100 owned both an automobile and a house.
 - To the nearest percent, what percent of people owned neither an automobile or a house?
 - To the nearest percent, what percent of people owned an automobile only?
- An investor has 100 shares total of two kinds of stock: Dot.Com and Big Blue Corp. Dot.Com sells for \$20 per share and Big Blue sells for \$50 per share. If the investor has \$2900 total in stock, how many shares of each does he own?
- In the 10 years prior to the international ban on ivory trade, the African elephant population declined by 690,000.
 - At what rate did the population decline, in elephants per year?
 - At the same rate of decline, how long would the remaining population, estimated at 897,000, have lasted had the ban not been implemented?
- Find the ordered pair (x,y) with the smallest possible y value that satisfies both $y \geq x^2$ and $y \geq x + 2$.
- A college student is going home for Thanksgiving. The student takes a helicopter from the dorm to the airport and a plane from the airport home. The helicopter averages 70 mph and the plane travels at a rate of 500 mph. The total time spent on the helicopter and plane was 2 hours and the total distance travelled by helicopter and plane is 785 miles. Find the distance from the dorm to the airport.
- Suppose a 3 letter code word is formed using the first 8 letters of the alphabet. How many code words can be formed if
 - no letter can be repeated?
 - adjacent letters cannot be alike?

SCHOOL NAME _____

SENIOR HIGH MATH LEAGUE
March 28,2000

GROUP II Emphasis on Algebra II

Answer Form A

SECTION I: ONE POINT EACH

SECTION II: TWO POINTS EACH

1. _____

1. (a) _____

2. _____

(b) _____

3. _____

(c) _____

4. _____

(d) _____

2. _____

3. _____

4. _____ hours

5. _____

6. _____

7. _____

8. _____

SCHOOL NAME _____

SENIOR HIGH MATH LEAGUE
March 28,2000

GROUP II Emphasis on Algebra II

Answer Form B

SECTION I: ONE POINT EACH

SECTION II: TWO POINTS EACH

1. _____ minutes

1. _____

2. (a) _____

2. (x,y)= _____

(b) _____

3. (a) _____

(b) _____

3. _____

4. Dot.Com _____

Big Blue _____

4. _____

5. (a) _____

(b) _____ years

6. (x,y)= _____

7. _____ miles

8. (a) _____

(b) _____